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Lightning Protection Volumes Around Trees: Cones, Rolling Spheres & Fractals

Dr. Kim D. Coder, Professor of Tree Biology & Health Care / University Hill Fellow
University of Georgia Warnell School of Forestry & Natural Resources

Examining sites and trees for potential installation of a lightning conduction system requires accurate calculations and measurements. Effectively minimizing lightning damage to trees and minimizing waste in purchasing lightning conduction materials requires careful planning. Tree protection professionals need to "rough-out" installations by estimating placement and amount of materials needed. It is critical to review and follow national and state standards and specifications for lightning conduction systems and for proper tree protection.

The three methods of determining air terminal and system effectiveness is by using the rolling sphere method, the cone (or angle) of protection method, or the fractal method. Note the rolling sphere and protective cone methods represent nearly the same protected volume beneath a tree. The fractal method suggests a larger protected volume for relatively short air terminal placements like in most trees.

Protection Cones

For relatively short structures like trees, lightning conduction systems have been historically designed using "cone of protection" concepts. A cone of protection is the idealized area (a right circular cone shape) beneath an air terminal within which most lightning damage should not occur, and is a simple way for visualizing the protected area under an air terminal. Figure 1. Remember, there is little protection from low current (<3kA) lightning strikes which comprise less than 0.5% of all ground strikes. In addition, the larger the cone angle used, the more likely is failure, especially with large current loads. (Uman 2008).

The protection cone (right circular cone) apex is centered upon the air terminal. The volume beneath the apex or within the cone is considered protected from most lightning strikes. The cone protection angle is one-half the apex angle. For increasing tree height, the cone protection angle (i.e. one-half the cone apex angle) narrows and becomes smaller in order to maintain the same protection effectiveness. For the same height of tree, the smaller half the apex angle becomes and the greater (or more sure) the protection level.

The size and shape of a cone of protection is a function of cone angle (a) in degrees and air terminal height (ht) in feet. Figure 2. A cone of protection model is valuable for defending space below an air terminal, and easy for visualizing and system design. A number of historic and current guidelines for lighting protection utilize a specified cone of protection. Figure 3.

Different cone angles provide different levels of protection. Figure 4. For example, if the air terminal height (ht) is one and the cone of protection angle (a) is 45^{o}, the ground distance radius of the cone of protection will be one multiplied by the air terminal height (1.0 X ht). Continuing with another example, if air terminal height (ht) is one and cone of protection angle (a) is 26°, then the ground distance radius of the cone of protection is 0.5 multiplied by the air terminal height (0.5 X ht). The effectiveness rating represents the middle of expected lightning strike peak currents.

Figure 5 provides a graph showing tree protection levels. For example, a cone angle (a) of 45° yields a 1:1 ratio of cone ground radius to air terminal height, and is considered to provide strong protection from lightning strike damage. A cone angle (a) of 63° yields a 2:1 ratio of cone ground radius to air terminal height, and is considered to provide moderate protection from lightning strike damage. In extreme protection situations, a cone angle (a) of 26° yields a 1:2 ratio of cone ground radius to air terminal height, and is considered to provide a high level of protection from lightning strike damage. (derived from USDoD, 1987).

Some cone angles have been cited in international, national, and state structural guidelines. Figure 6 shows cone angles used to attain specific lightning protection effectiveness for given tree heights.

Rolling Sphere

The rolling sphere method is another means to visualize protected volume below an air terminal. The rolling sphere method for determining the protected area beneath an air terminal is used for many types of lightning protection systems. The lightning strike distance used for design and installation of a tree lightning conduction system is represented by the radius of the sphere. Usually a standard sphere radius is used representing the effectiveness of any system installed. The volume below and outside the sphere is considered protected from a majority of lightning strikes, but small and extremely large current lightning strikes would not be defended against.

Structural protection volumes below air terminals around buildings and trees have been delineated by the rolling sphere method. Figure 7. As a sphere of a given radius is rolled over and around any structure, the sphere touches the ground and structure's air terminal. Beneath the edge of the sphere (sphere surface) between contact points defines the limit of the protection area. Figure 8. Various levels of structural protection are reached by using spheres with different radii (sphere size). Figure 9. Smaller radius spheres are used to provide a greater level of protection. Figure 10. For example, a 150 feet arc radius would represent a protection area for ~91% of lightning strikes which include peak currents between 10-100kA. (IEC 1992; Volland 1995)

For trees, the 150 feet arc radius rolling sphere is normally used in North America. Trees taller than rolling sphere radius selected would need additional protection to maintain the protection effectiveness. Figure 11.

Around the world, different rolling sphere radii are placed into standards and installation guides, usually associated with building or structure protection, not trees. Figure 12 shows the two most common tree associated radii and the horizontal extent of their protection zone based upon a 50 feet air terminal height. International protection standards using the rolling sphere method give a range of protection effectiveness provided in peak current (kA) range coverage and rolling sphere radius in feet. Many tree installations are made at the class III level trying to meet greater than a 91% protection effectiveness.

Figure 13 provides estimated tree protection zone distances and areas below a single air terminal at a given height. This tree protection area is based upon a standard 150 feet arc radius line. Protection area radius, diameter, and area measures are given for the ground beneath an air terminal in a tree using the formulae below:

protection area radius =

$$((\text{ArcRad})^2 - (\text{ArcRad} - \text{air terminal height})^2)^{0.5}$$

protection area diameter =

$$2 \times (((\text{ArcRad})^2 - (\text{ArcRad} - \text{air terminal height})^2)^{0.5})$$

protected ground area =

$$(3.142) \times [((\text{ArcRad})^2 - (\text{ArcRad} - \text{air terminal height})^2)^{0.5}]^2$$

ArcRad = rolling sphere radius in feet

For example, a tree with a single air terminal installed at 100 feet above the ground would have a protection area radius below the air terminal following a 150 feet arc radius sphere edge which touches the ground at 141 feet away from the stem center. Diameter of the protection area in this example is 282 feet (2X radius) centered on the tree stem. The protected ground area would be 62,831 square feet of soil surface. Figure 14 graphically illustrates the example. Remember lightning conduction systems generate a three-dimensional protection volume, requiring design considerations from all views.

Visualizing Rolling Protection

When looking at a tree, it is important to visualize the protected volume of space above ground and beneath any air terminal. How much of the tree is covered by any protected space? Figure 15 provides the estimated horizontal distance away from the stem a protection area extends on the ground underneath a single, tree-centered air terminal placed at a given height within a tree for a series of different vertical heights. For example, a tree with an air terminal positioned at 80 feet in height would have an edge of its protection area at 50 feet above the ground located 20 feet radially away from the stem center. These values were generated with the following formula:

Horizontal Radial Distance to the edge of the protection area from tree stem center =

$$(\text{PZr}) - [(\text{ArcRad})^2 - (\text{ArcRad} - \text{Vht})^2)^{0.5}]$$

PZr = protection zone radius on the ground
Vht = vertical height at some distance from stem center
ArcRad = rolling sphere radius in feet

Figure 16 provides the estimated vertical height of the protection area beneath a single, tree-centered air terminal at a given height for a series of horizontal radial distances. For example, a tree with a single crown-centered air terminal positioned at 80 feet in height would have the edge of the protection area located 30 feet radially away from the stem center at a height of 40 feet above the ground. These values were generated with the following formula:

Vertical Height to the edge of the protection area from ground level =

$$\text{ArcRad} - [(\text{ArcRad})^2 - (\text{PZr} - \text{Hrd})^2]^{0.5}$$

PZr = protection zone radius on the ground
HRd = horizontal radial distance from stem center at some height
ArcRad = rolling sphere radius in feet

Figure 17 shows how a 150 feet arc radius sphere would be used to minimize risk of a lightning strike on a tree. One-half the crown for a 100 feet tall tree, with a 100 feet height air terminal, is shown at the lower left. The tree diagram has two branches 60 feet long. In this case, the 150 arc radius sphere touches both the ground and the top of the tree at its air terminal location. The area below the line and outside the 150 arc radius sphere dotted line would be within the protection area for 91% of lightning strikes.

For example, a tree with a 100 feet tall air terminal placement would have a protection area which is 23 feet above the ground 60 feet away from the tree stem center, and which is 49 feet above the ground 30 feet away from the tree stem center. This places branch B within the protection area and the furthest portion of branch A outside the protection zone.

Figure 18 provides an idealized two-dimensional view of a tree with an air terminal. The protection area would be below the dotted 150 feet arc radius lines shown. Most trees can be protected adequately with a single air terminal placed high in the tree and centered over the crown. Remember a tree protection system covers a three-dimensional volume which is represented here by two-dimensional drawings.

Figure 19 is a worksheet for drawing out a rough view of the above ground portion of a tree lightning conduction system Use this worksheet to draw the side view outline of a tree to be protected using measures from the field. The dotted lines on the worksheet can be used for any tree to place an air terminal with a height of 120 feet or less. This worksheet uses a 150 feet arc radius rolling sphere value in determining the tree protection area.

Fractal Space

The newest way to design and visualize tree protection systems uses fractal analysis. This system is significantly different from the two preceding methods. Figure 20. For relatively short structures like trees, fractal analysis suggests a much greater volume of protected space.

Citation:
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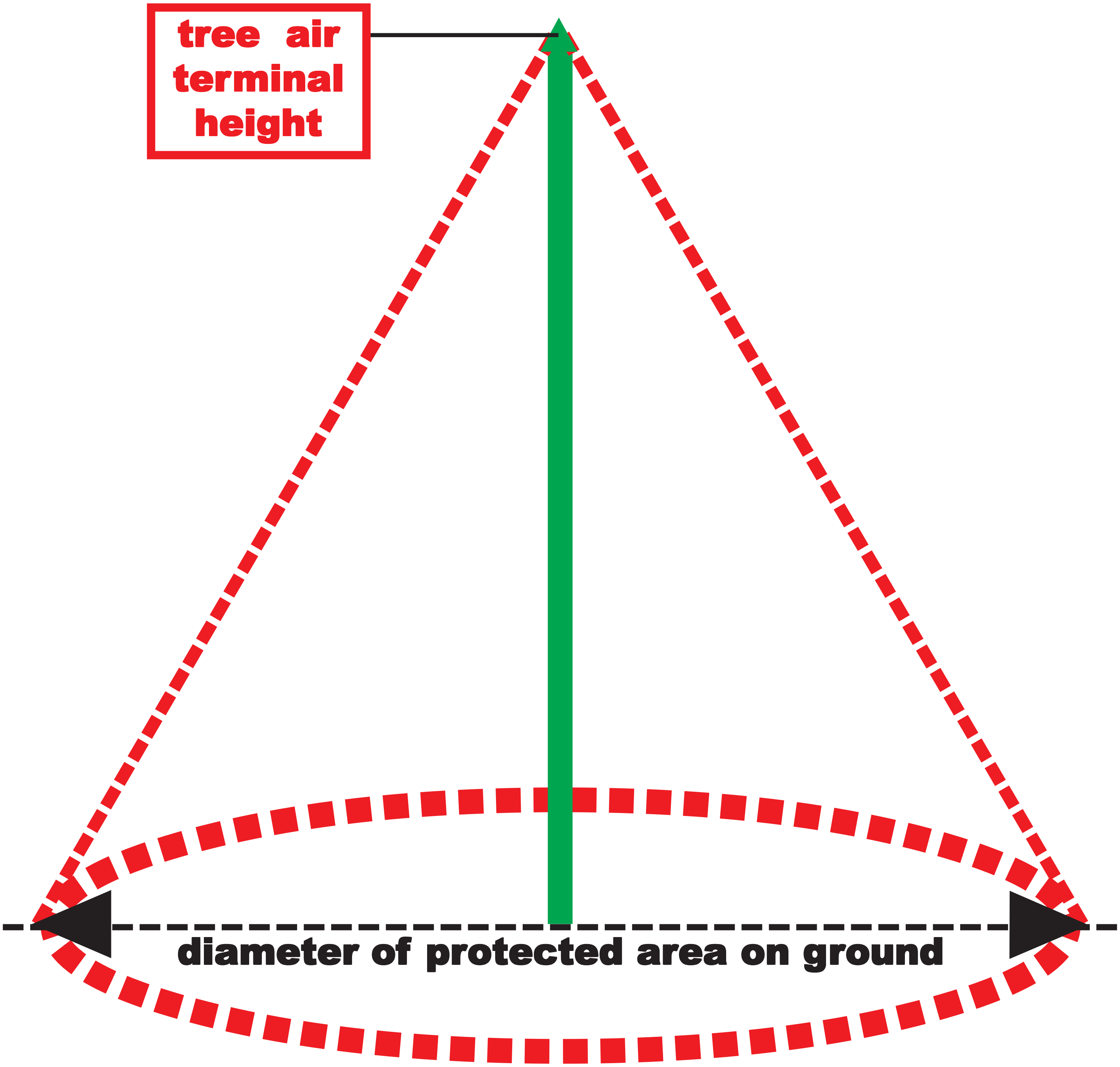

Figure 1: Description of a right circular cone shape whose apex is the air terminal of a lightning conduction system.
(Bouquegneau 2010)

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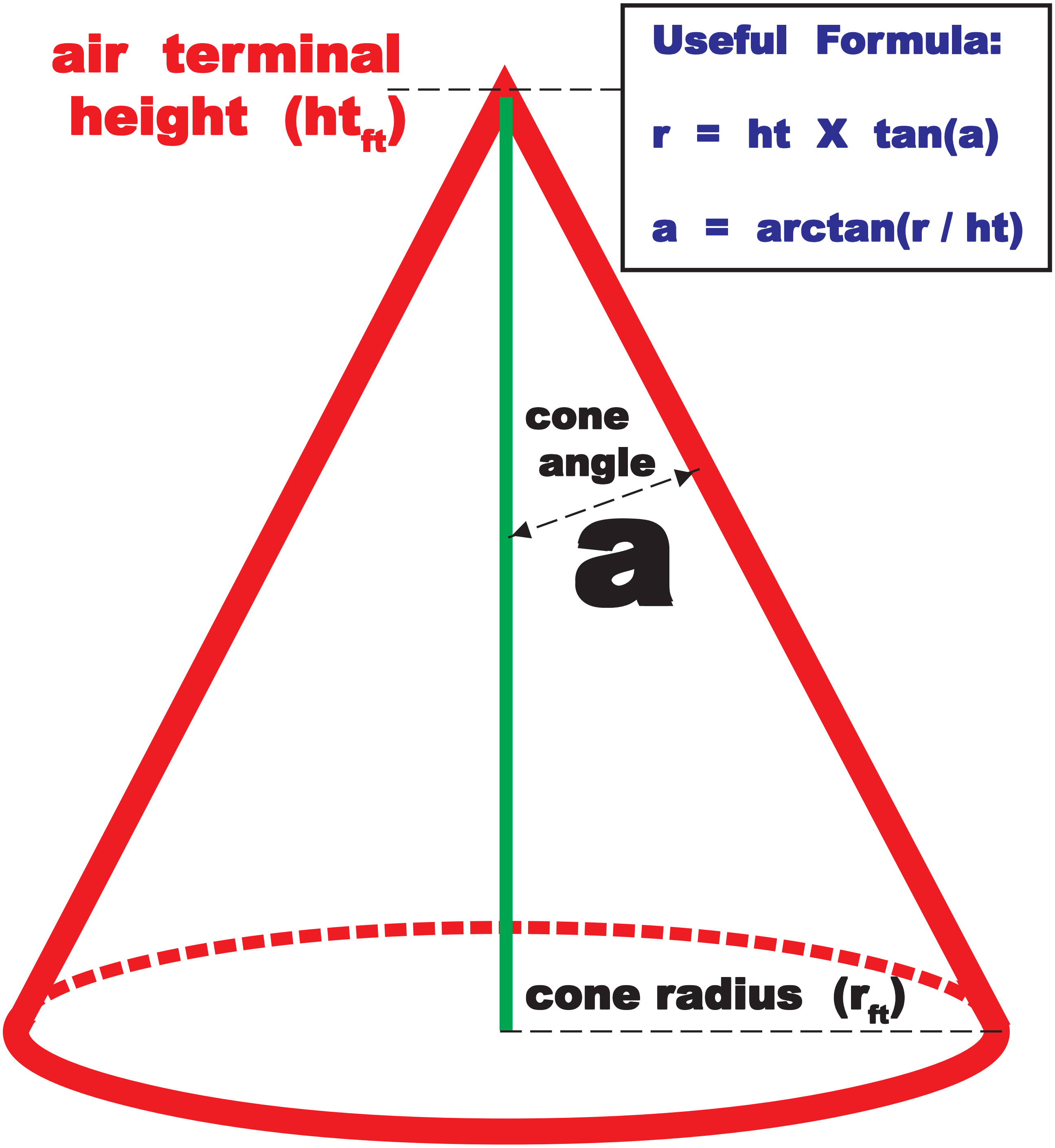

Figure 2: Diagram of tree lightning conduction system "cone of protection" dimensions beneath an air terminal. Cone angle "a" is 1/2 apex angle in degrees.

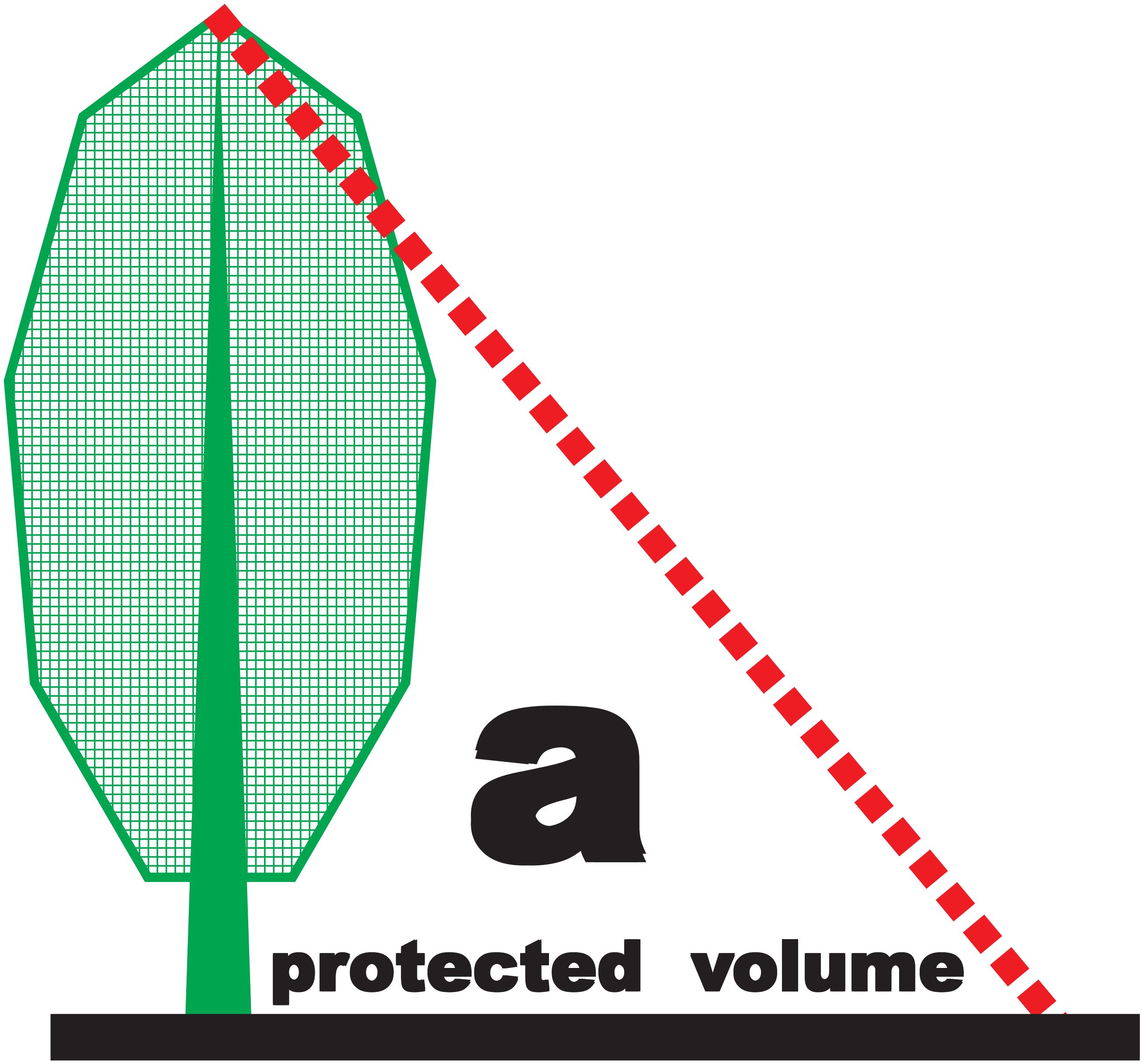

Figure 3: Two-dimensional side view of a protection cone (one-half of a right circular cone) beneath an air terminal in a tree. "a" is the protection cone angle (1/2 apex angle) in degrees. (Bouquegneau 2010)

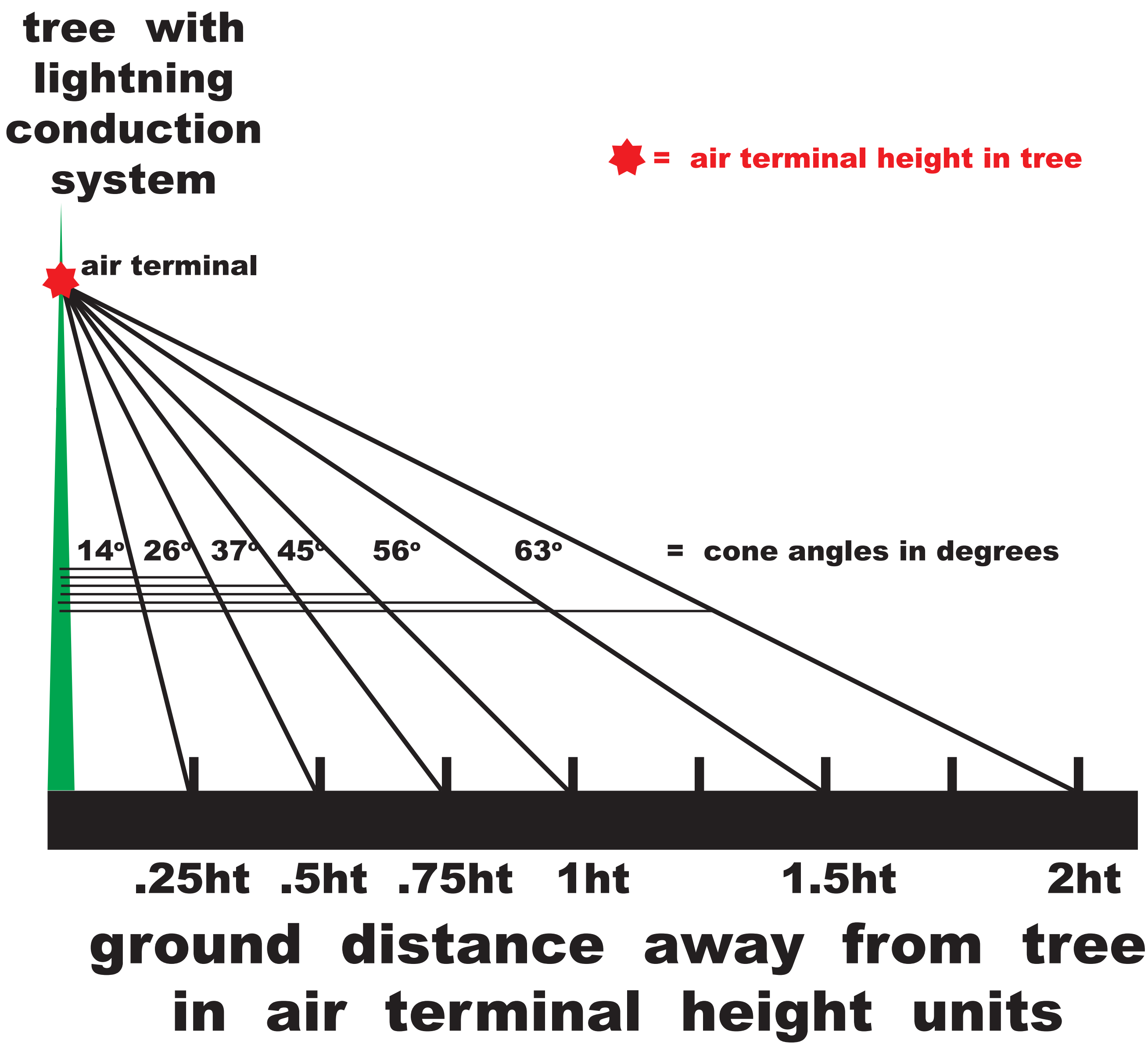

Figure 4: Two dimensional side view for series of three dimensional "cones of protection" beneath a lightning conduction system air terminal. The ground distance and cone angle are shown. (derived from USDoD, 1987)

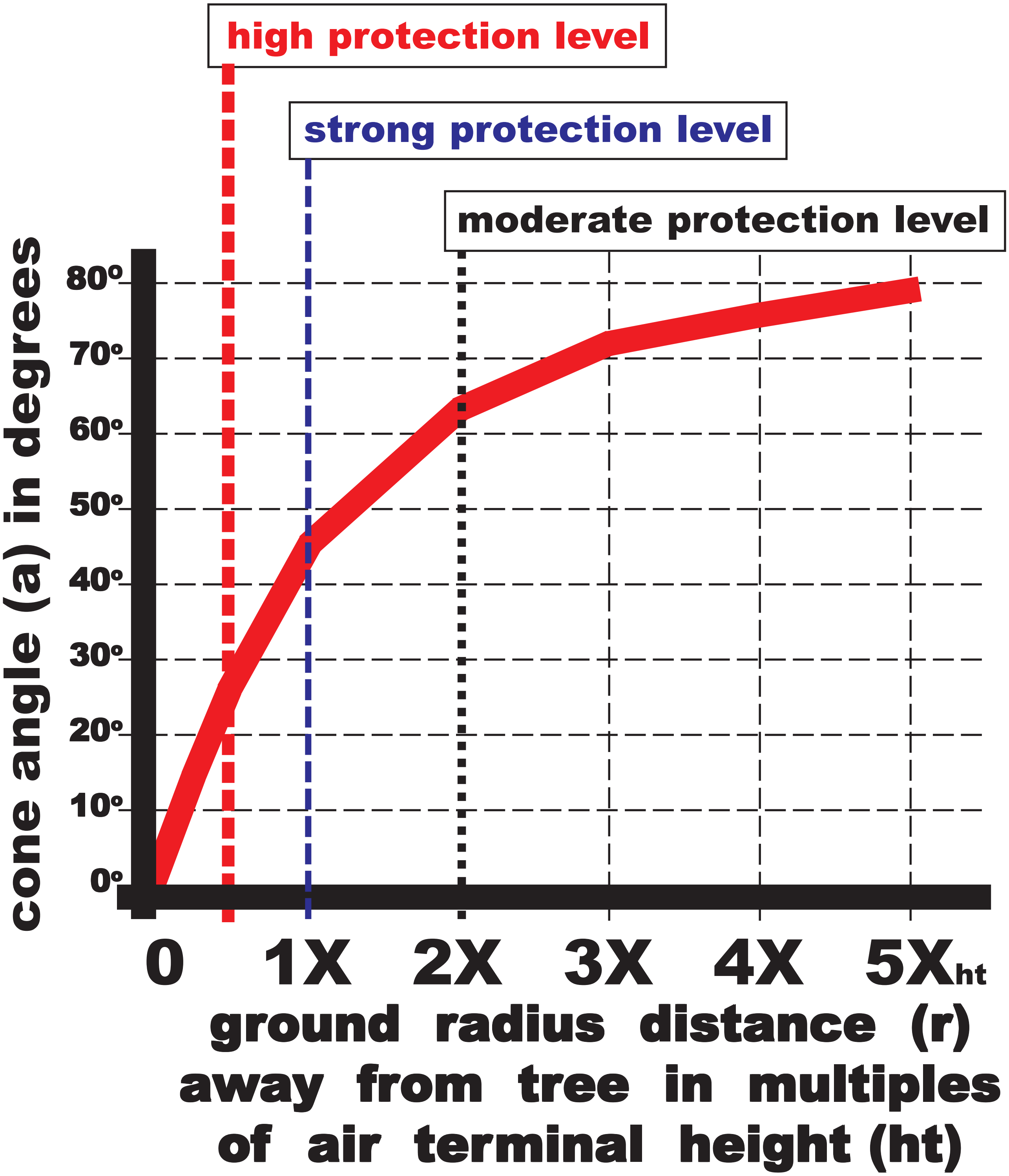

Figure 5: Graph of cone angle (a) and ground radius distance (r) using cone height (ht) units. (i.e. 2X ground radius = two times the cone height).
(derived from USDoD, 1987)

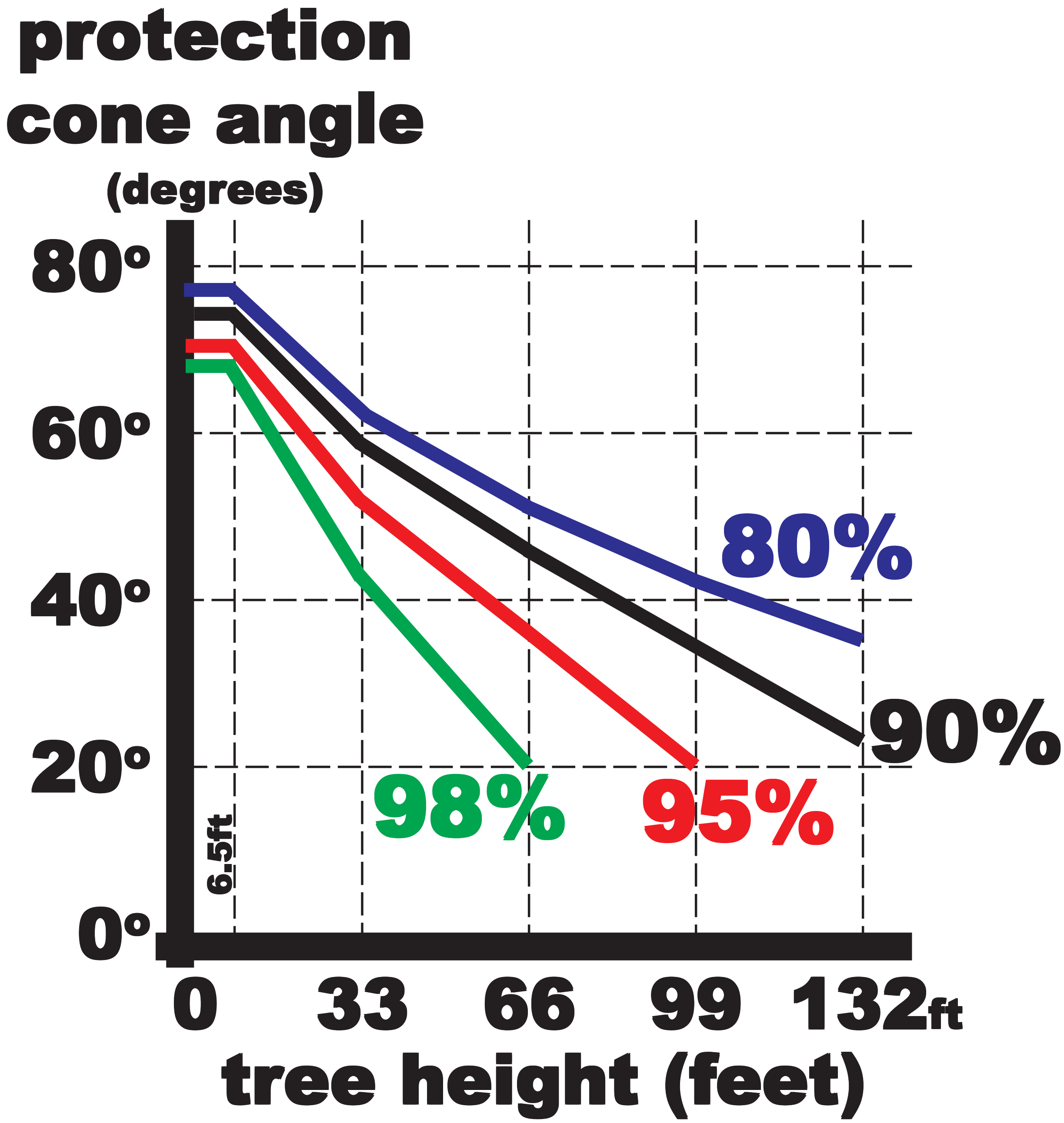

Figure 6: Protection cone angle for various tree heights (in feet) and for different levels of lightning protection (in percent). (Bouquegneau 2010)

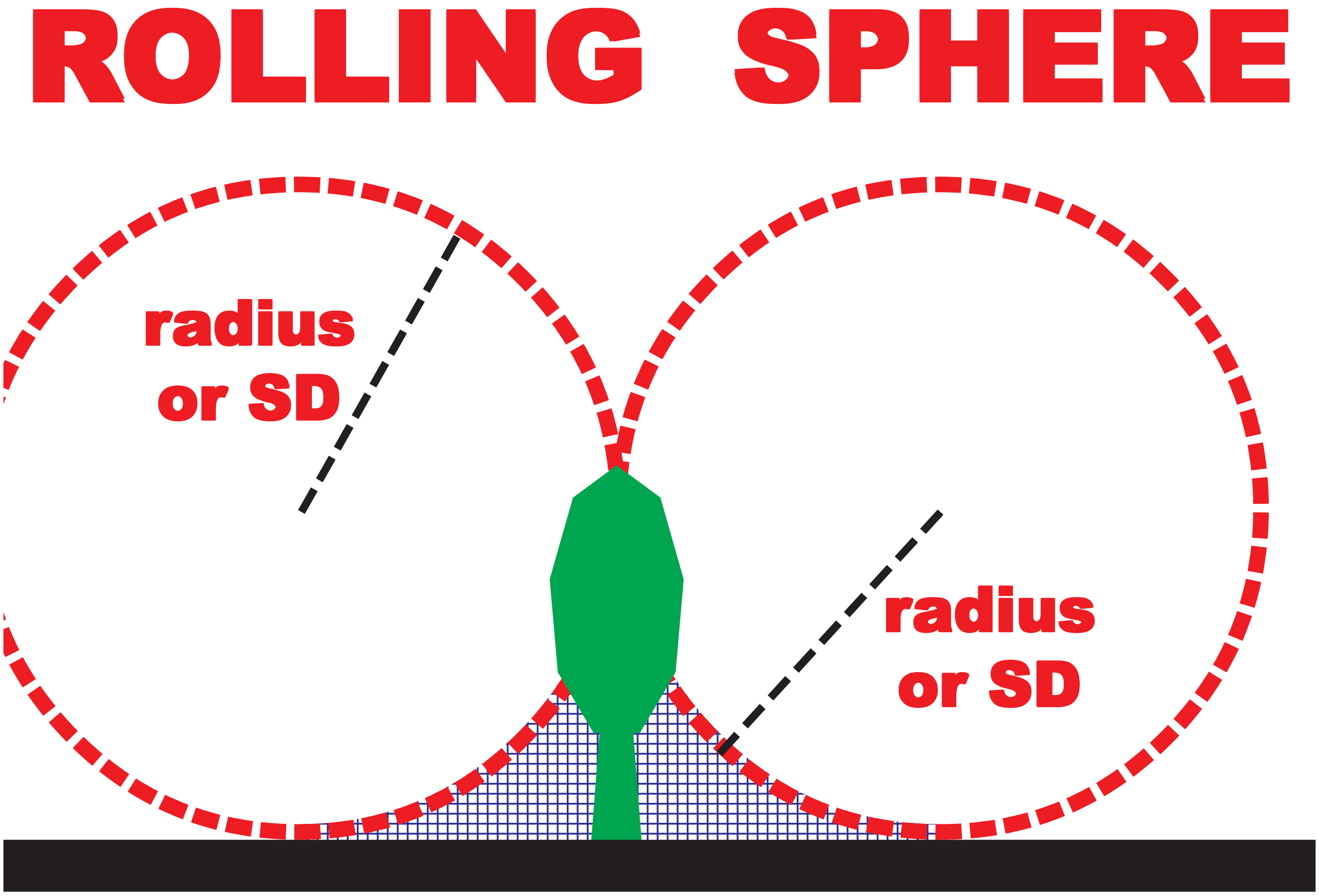

Figure 7: Rolling sphere method for determining volume protected beneath (shaded area) an air terminal of a lightning conduction system in a tree. (Rakov 2012)

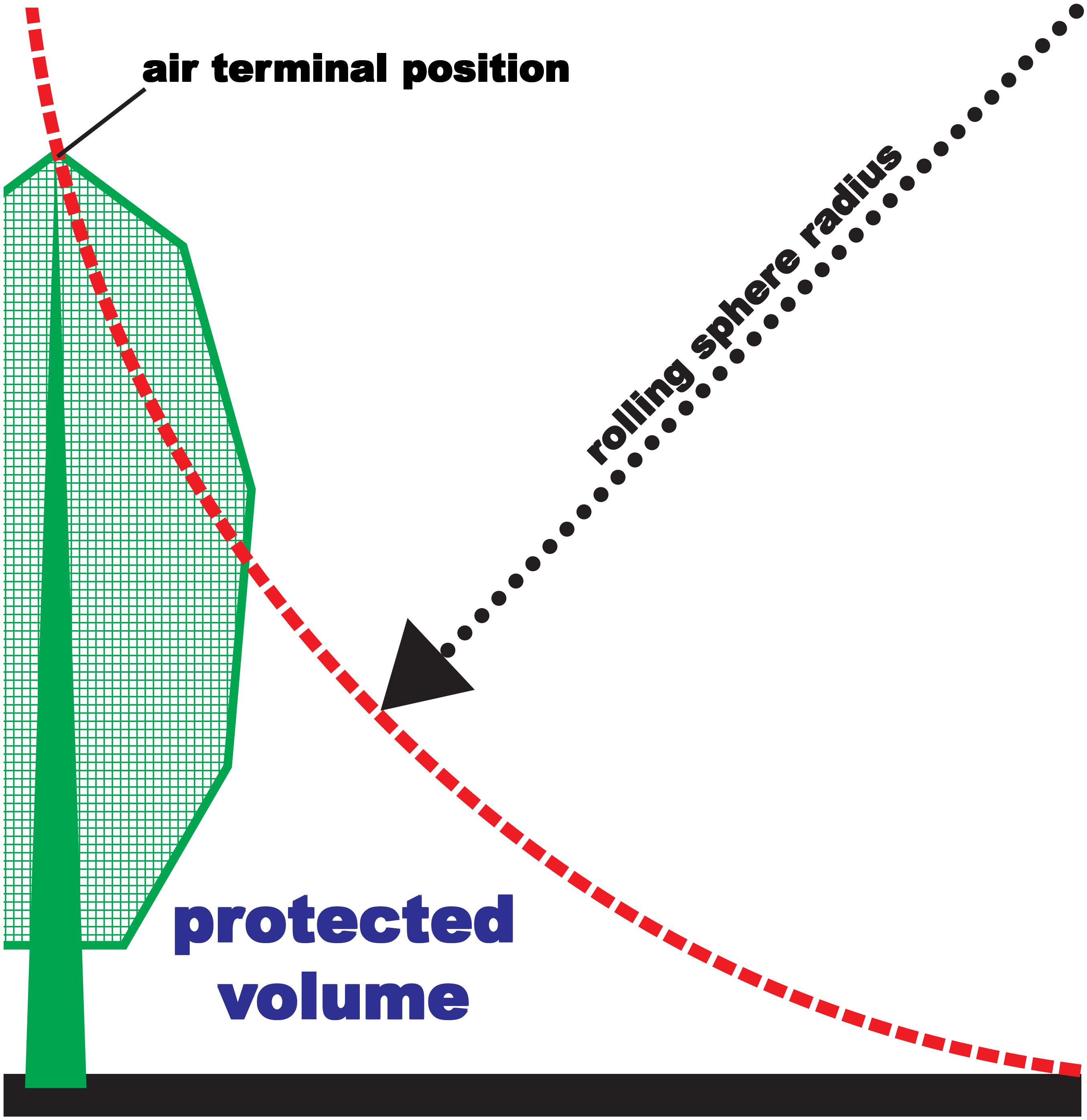

Figure 8: Two-dimensional side view of a volume of space around a tree beneath an air terminal determined by the rolling sphere method considered protected for some level of lightning strikes. (Bouquegneau 2010)

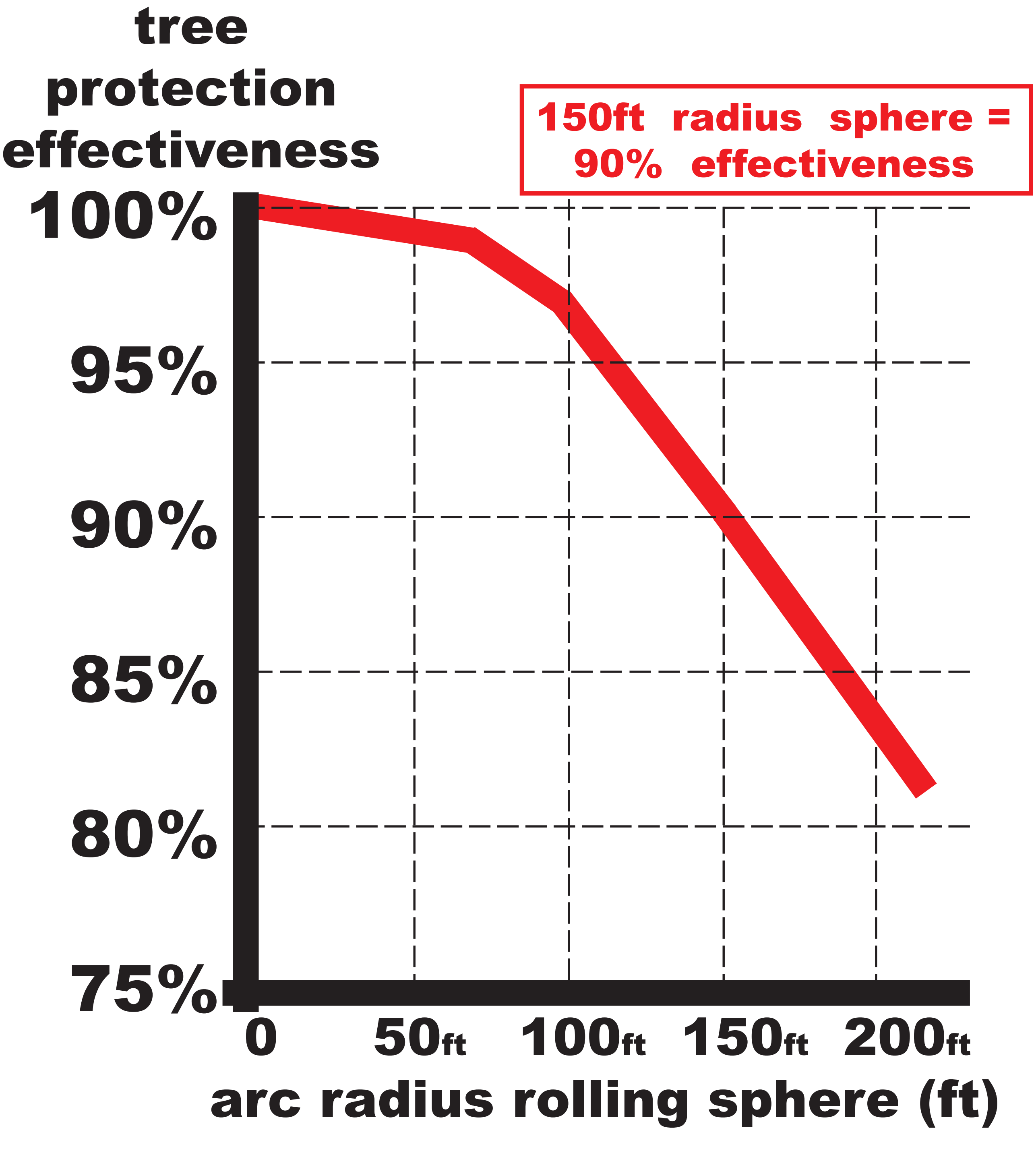

Figure 9: Protection effectiveness using the rolling sphere method in determining protection area below air terminal in a tree. (derived from IEC, 1992; Volland 1995)

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International Standard (IEC)

Classes of Lightning Protection

class	protection value	current	rolling sphere radius
I	99%	3-200kA	~ 66ft
II	97%	5-150kA	~100ft
III	91%	10-100kA	~150ft
IV	84%	15-100kA	~200ft

Figure 10: International lightning protection standrad values for using rolling sphere method. (Bouquegneau 2010)

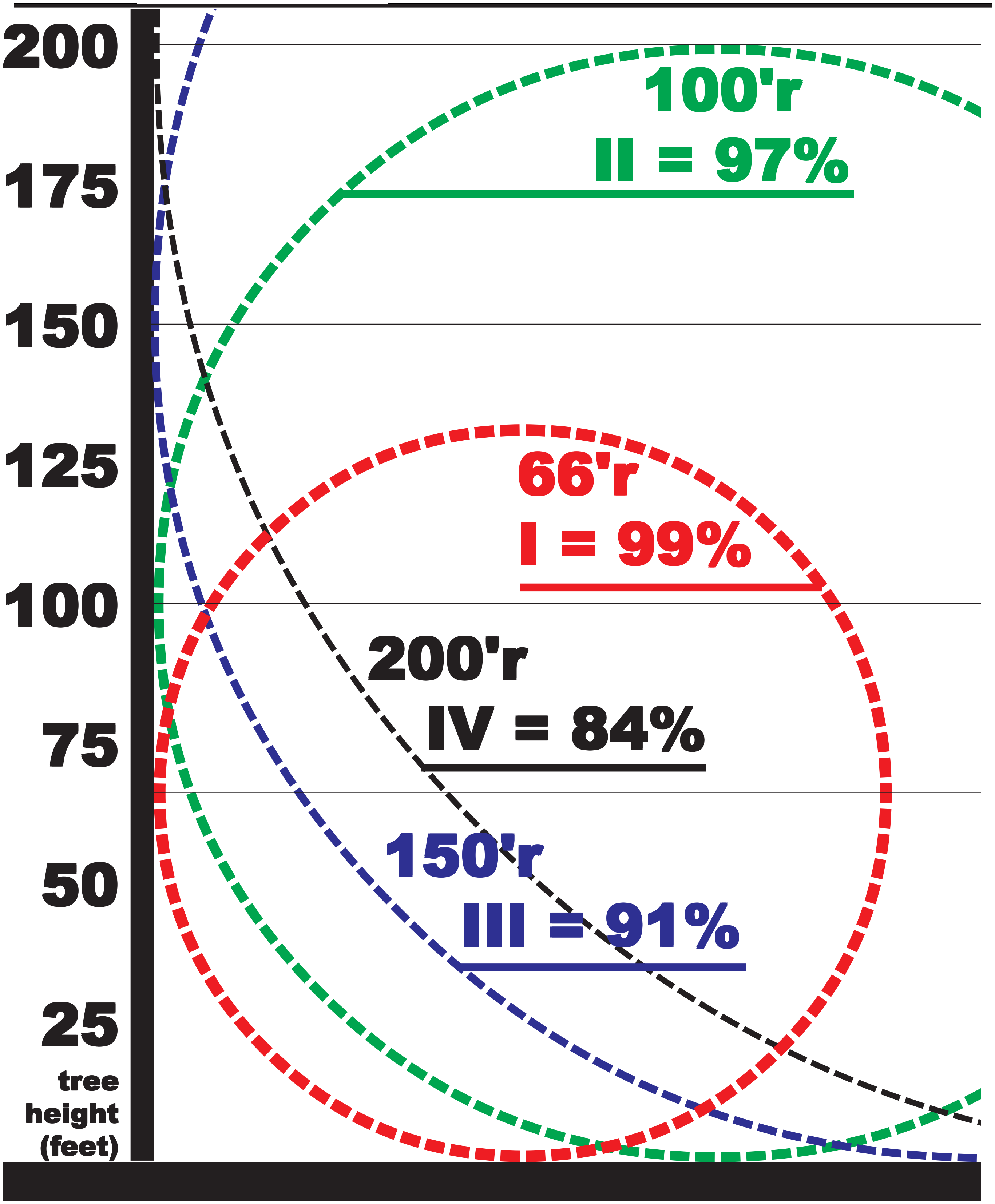


Figure 11: Comparing the relative sizes of rolling sphere radius values for tree lightning protection. (Bouquegneau 2010)

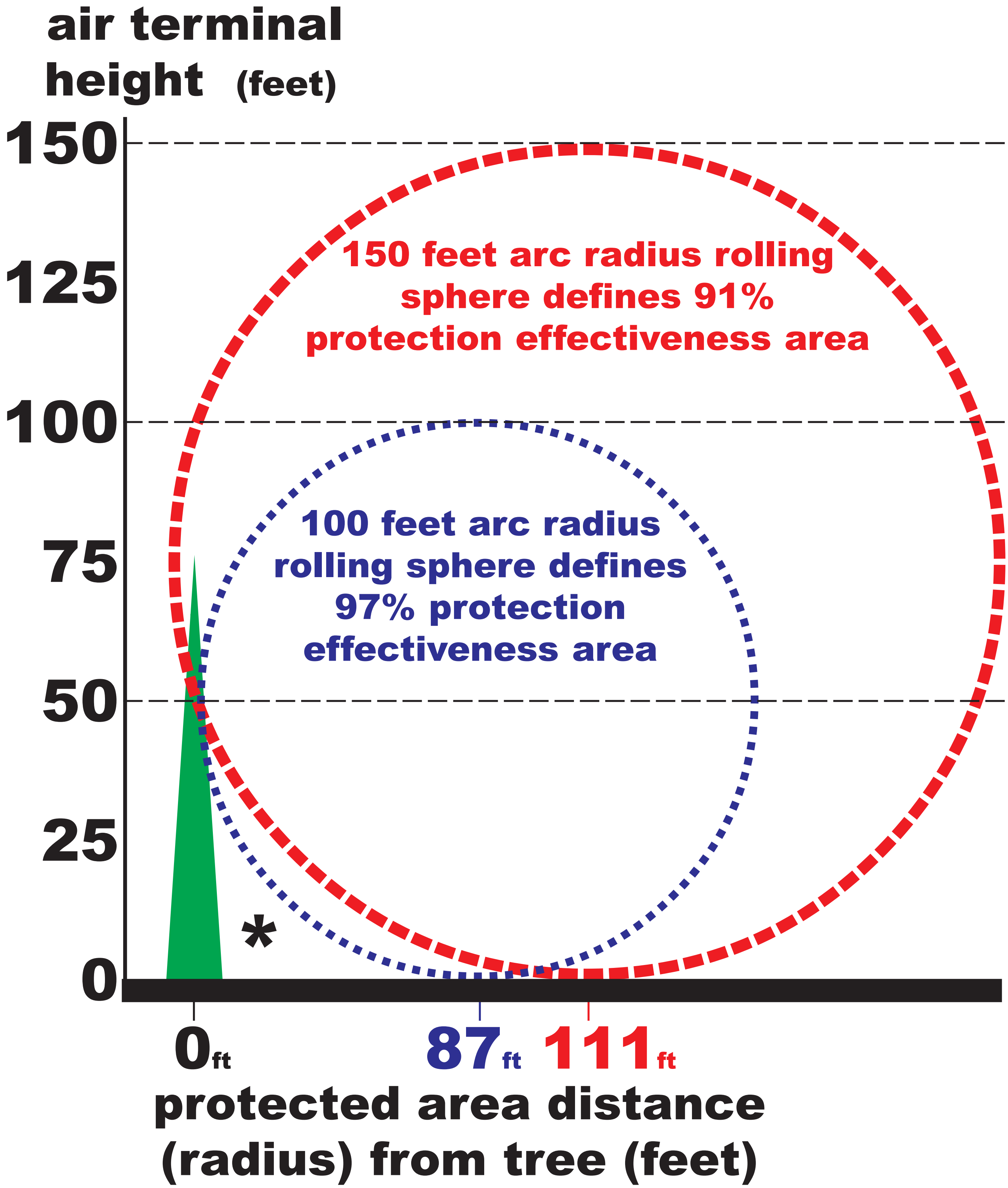


Figure 12: Rolling sphere measures for estimating protection area under a tree air terminal placed 50 ft above ground.

air terminal height (feet)	protection area radius (feet)	protection area diameter (feet)	protected ground area (sq. ft.)
10 ft.	53 ft.	107 ft.	9,110 sq.ft.
20	74	149	17,592
30	90	180	25,446
40	101	203	32,672
50	111	223	39,269
60	120	240	45,238
70	126	253	50,579
80	132	265	55,291
90	137	274	59,376
100	141	282	62,831
120	146	293	67,858
140	149	299	70,371

Figure 13: Estimated protection distances (in feet) and area (in square feet) measured on ground below single air terminal which is installed at a given height (in feet) in a tree (using an 150 feet arc radius protection zone).

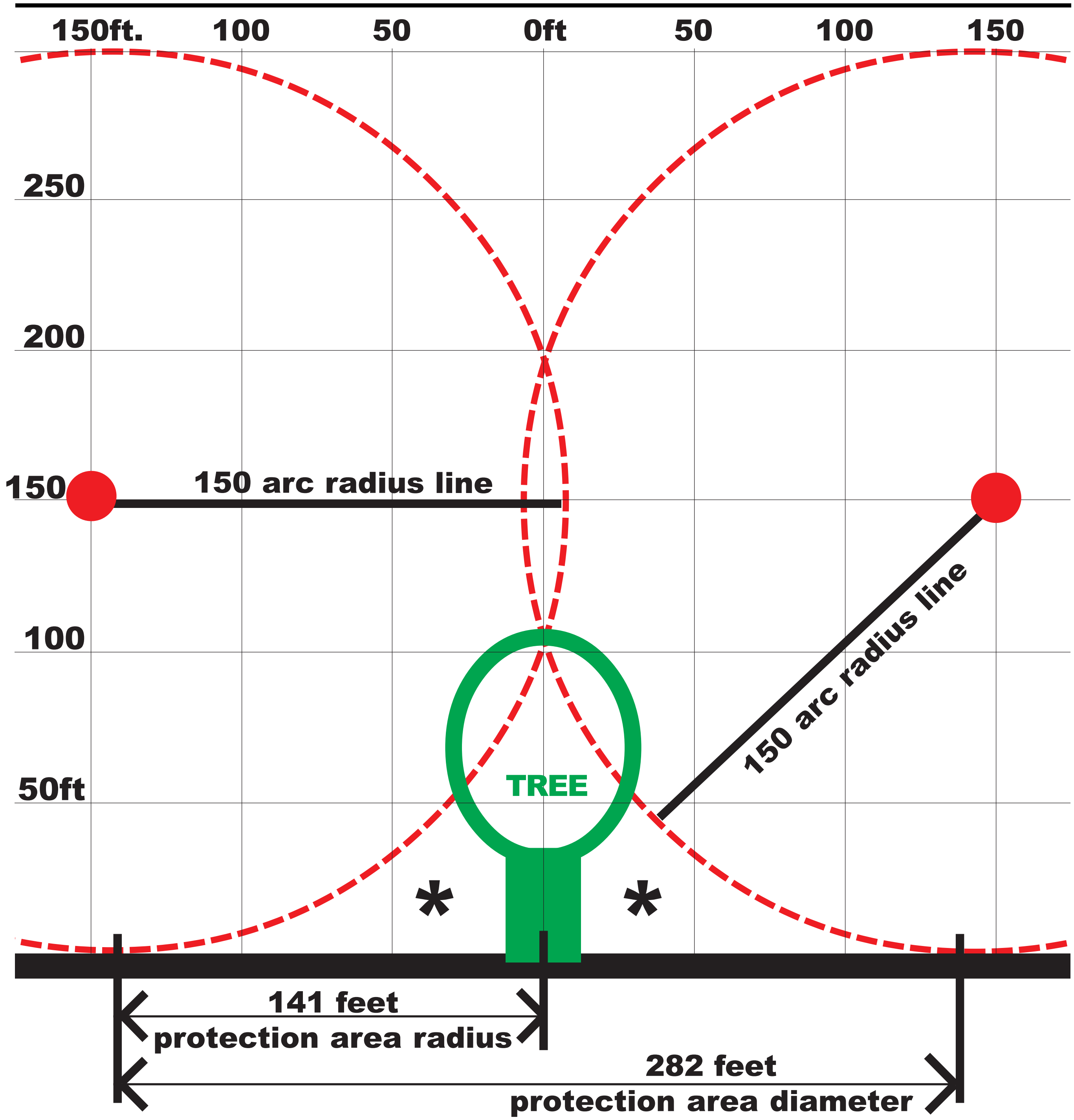


Figure 14: Representation of protection area (*) beneath air terminal in a tree. When air terminal is positioned at 100 feet height, protection area edge follows an 150 feet arc radius line that touches ground at 141 feet away from tree base.

air terminal height (feet)	vertical distances above ground (feet)											
	10	20	30	40	50	60	70	80	90	100	120	140
10	0ft											
20	20	0										
30	36	15	0									
40	47	26	11	0								
50	57	36	21	9	0							
60	66	45	30	18	8	0						
70	72	51	36	24	14	6	0					
80	78	57	42	30	20	12	5	0				
90	83	62	47	35	25	17	10	4	0			
100	87	66	51	39	29	21	14	8	3	0		
120	92	71	56	44	34	26	19	13	8	4	0	
140	95	74	59	47	37	29	22	16	11	7	2	0

Figure 15: Estimated horizontal extent of the protection area (in feet) beneath a single air terminal at a given height (in feet) for a series of heights. (using 150 feet arc radius)

air terminal height (feet)	horizontal radial distances from stem center (feet)											
	10	20	30	40	50	60	70	80	90	100	120	140
10	6ft	3	1	0								
20	14	10	6	3	1	0						
30	23	17	12	8	5	3	1	0				
40	30	23	17	12	8	5	3	1	0			
50	39	30	23	17	12	8	5	3	1	0		
60	48	38	30	23	17	12	8	5	3	1	0	
70	54	43	34	27	20	15	10	7	4	2	0	
80	62	50	40	31	24	18	13	9	6	3	1	0
90	70	56	44	35	27	21	15	11	7	4	2	0
100	76	61	49	39	30	23	17	12	8	5	3	1
120	86	68	54	43	34	27	20	15	10	7	4	2
140	93	73	58	46	37	29	22	16	12	8	5	2

Figure 16: Estimated vertical height of protection area (in feet) beneath a single air terminal at a given height (in feet) for a series of horizontal radial distances. (150 feet arc radius).

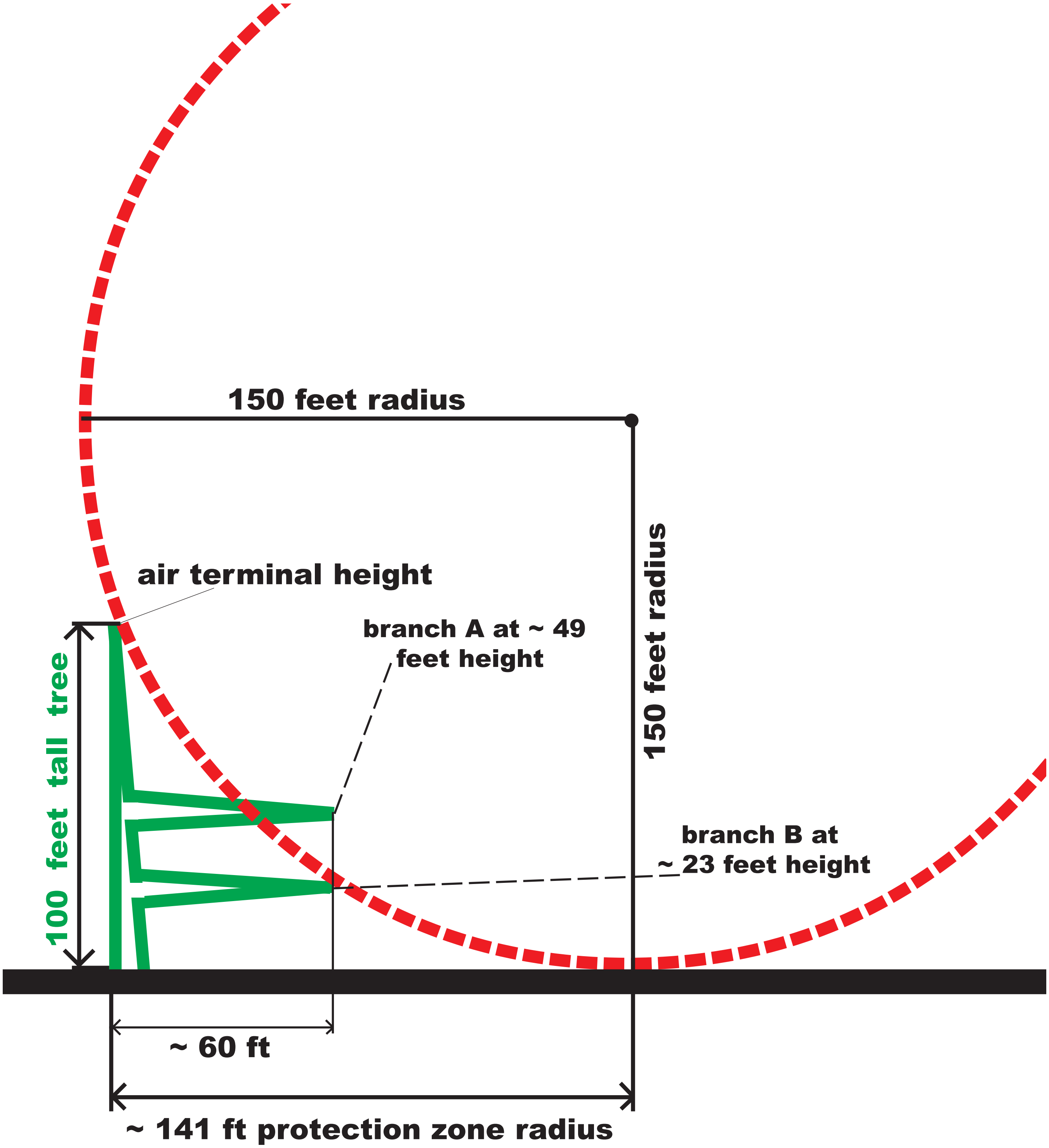


Figure 17: Description of a tree protection area based on 150 feet arc radius delineation line (dotted circle).

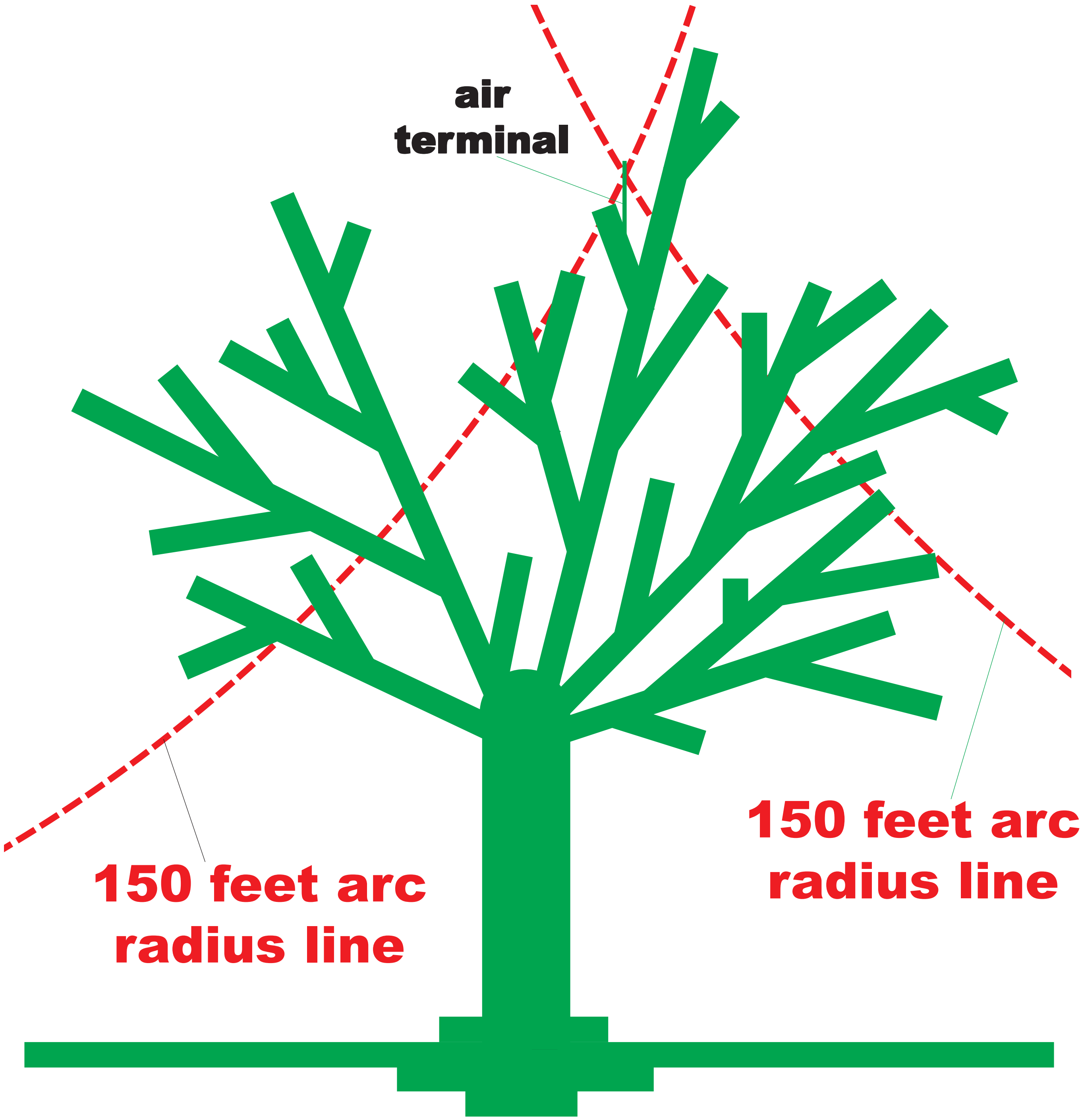
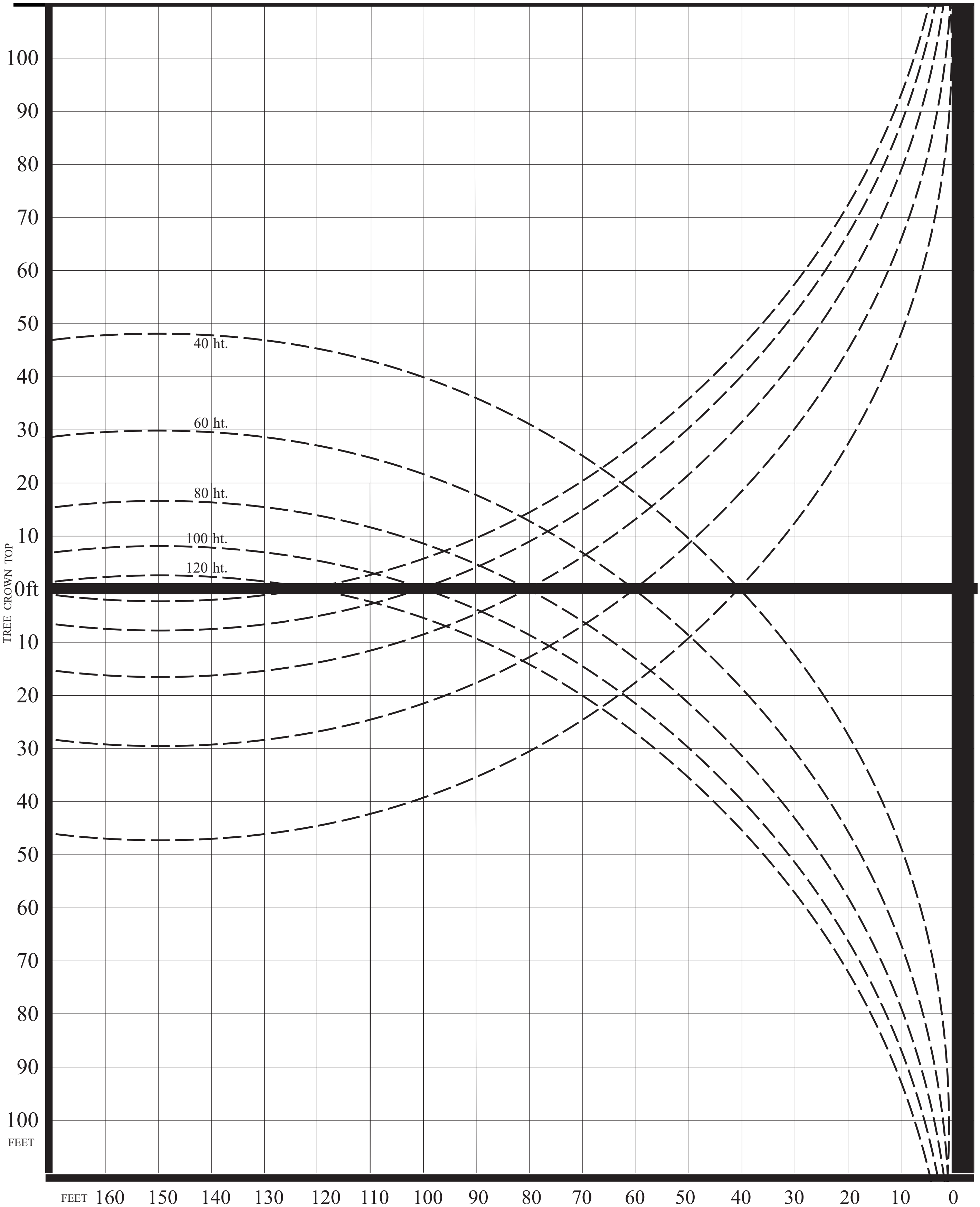


Figure 18: Estimated protection zone beneath air terminal.

Figure 19: Protection system design worksheet for estimating distances, protection areas, and materials. (150 feet arc radius circle)



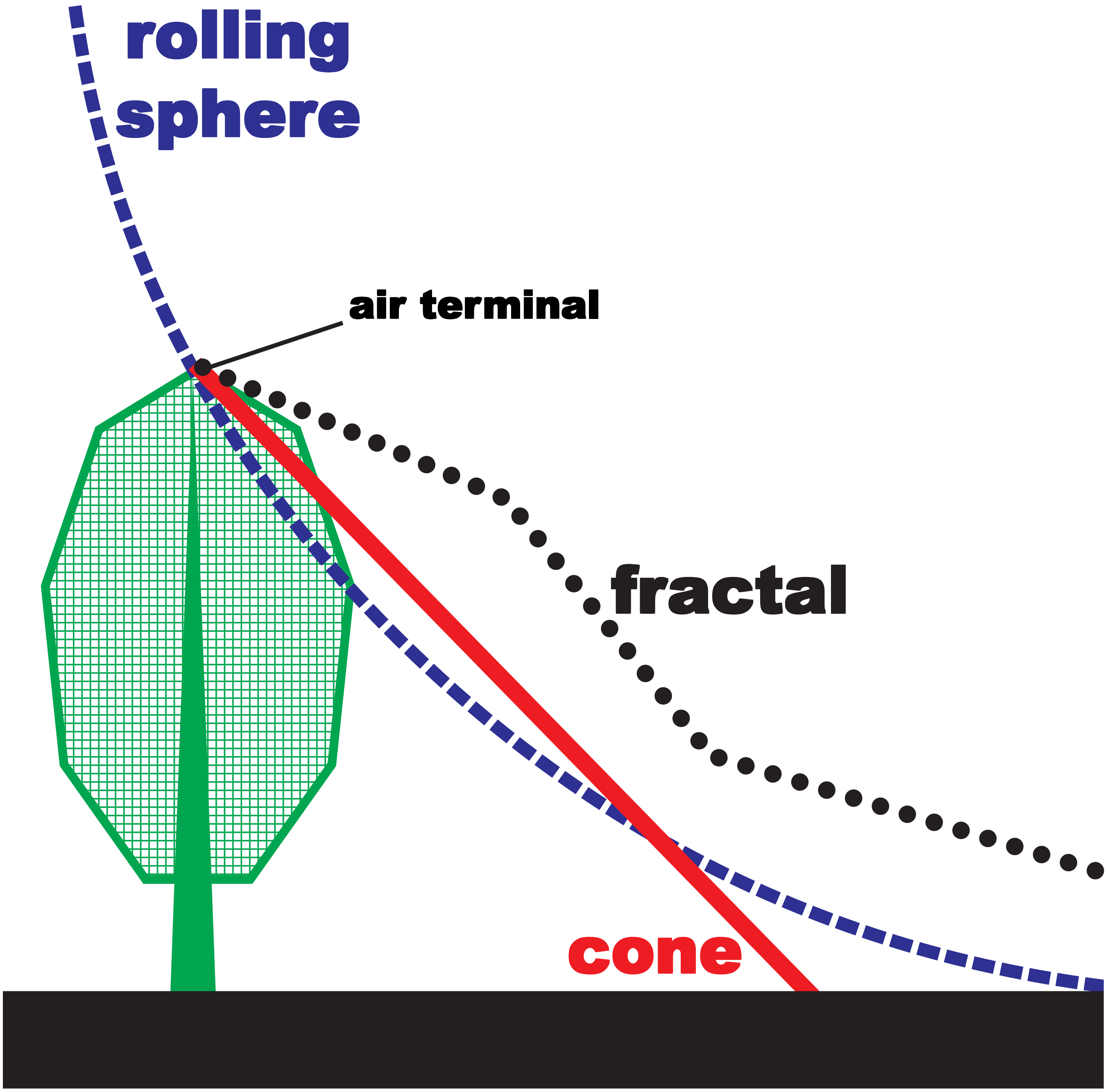


Figure 20: Comparison of cone, rolling sphere, and fractal protection zones (i.e. one-half zone) for tall, isolated tree. Protected areas are below lines. (derived from Zhang et.al. 2009)